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RP-003-1015044

Seat No. _______

B. Sc. (Sem. V) (CBCS) Examination

February - 2019

S-503 : Statistical Inference

(New Course)

Faculty Code : 003

Subject Code : 1015044

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (i) All question carry equal marks.

(ii) Student can use their own scientific calculator.

1 (a) Give the answer of following question : **4**

(1) A single value of an estimator for a population parameter θ is called its _____ estimate.

(2) If an estimator T_n converges in probability to the parametric function $\tau(\theta), T_n$ is said to be a _____ estimator.

(3) If the expected value of an estimator T_n is equal to the value of the parameter θ, T_n is said to be ______ estimator of θ.

(4) If T_n is an estimator of a parametric function $\tau(\theta)$, the mean square error of T_n is equal to ____.

(b) Write any one : **2**

(1) Obtain an unbiased estimator of θ for $f(x,\theta)=\frac{1}{\theta}$; where $0 \le x \le \theta$.

(2) Obtain sufficient estimator of θ for $f(x,\theta)=\frac{1}{\theta}e^{-\left(\frac{x}{\theta}\right)}$; where $0 \le x \le \infty$.

(c) Write any one : **3**

(1) Obtain consistent estimator of p for binomial distribution.

(2) If $t''=\frac{1}{2}(t+t')$ where t and t' is the most efficient estimator with variance v then prove that $Var(t'')=\frac{1}{2}v\left(1+\sqrt{e}\right)$.

(d) Write any one : **5**

(1) If $x \sim N\left(\mu,\sigma^2\right)$ and μ is known then obtain unbiased estimator of σ.

(2) If T_1 and T_2 be two unbiased estimator of θ with variance σ_1^2, σ_2^2 and correlation ρ, what is the best unbiased linear combination of T_1 and T_2 and what is the variance of such combination ?

2 (a) Give the answer of the following question : **4**

(1) If T_n is an estimator of a parameter θ of the density $f(x;\theta)$ the quantity $E\left[\frac{\partial}{\partial\theta}\log f(x;\theta)\right]^2$ is called the ______.

(2) An estimator of $v_\theta(T_n)$ which attains lower bound for all θ is known as _____.

(3) An unbiased and complete statistic is a _____ estimator provided MVUE exists.

(4) For discrete variable Crammer-Rao inequality _____.

(b) Write any one : **2**

(1) Find Cramer-Rao lower bound of variance of unbiased estimator of θ for $f(x,\theta)\theta = e^{-x\theta}$.

(2) Define minimum variance bound estimator.

(c) Write any one : **3**

(1) Obtain MVUE and MVBE of θ for Poisson distribution.

(2) Independent observation $x_1, x_2, x_3, ..., x_n$ taken from the following density function.

$$f(x,\theta) = \frac{1}{\theta} e^{-\left(\frac{x}{\theta}\right)}; \text{ where } 0 \le x \le \infty$$

Find the Cramer Rao lower bound for variance of unbiased estimator of θ.

(d) Write any one : **5**

(1) State and prove Cramer-Rao inequality.

(2) Obtain MVUE and MVBE of σ^2 for normal distribution.

3 (a) Give the answer of following question : **4**

(1) If a random sample $x_1, x_2, x_3, ..., x_n$ is drawn from a population $N(\mu, \sigma^2)$, the maximum likelihood estimate of σ^2 is _____.

(2) Maximum likelihood estimate of the parameter θ of the distribution $f(x,\theta) = \frac{1}{2} e^{-|x-\theta|}$ is _____.

(3) For a Gama (x, α, λ) distribution with λ known, the maximum likelihood estimate of α is _____.

(4) For a rectangular distribution $\frac{1}{(\beta - \alpha)}$, the maximum likelihood estimates of α and β are _____ and _____ respectively.

(b) Write any one : **2**

(1) Obtain likelihood function of negative binomial distribution.

(2) Estimate parameter θ by the method of moment for the following distribution $f(x, \theta) = \frac{1}{\theta} x$; where $0 \le x \le 1$.

(c) Write any one : **3**

(1) Obtain MLE of θ for the distribution

$f(x, \theta) = (\theta + 1) x^{\theta}$; where $0 \le x \le 1$.

(2) Estimate parameters α and β by the method of moment for the following distribution.

$$f(x, \alpha, \beta) = \frac{\alpha^{\beta}}{\overline{|\beta}} e^{-\alpha x} x^{\beta - 1}; \text{ where } x \ge 0, \alpha > 0.$$

(d) Write any one : **5**

(1) Obtain MLE of α and λ for the following distribution.

$$f(x, \alpha, \lambda) = \frac{1}{\overline{|\lambda}} \left(\frac{\lambda}{\alpha}\right)^{\lambda} e^{-\left(\frac{\lambda}{\alpha}\right)x} x^{\lambda - 1}; \text{ where } 0 \le x \le \infty, \ \lambda > 0$$

where $\varphi(\lambda) = \frac{\partial}{\partial \lambda} \log \overline{|\lambda} = \log \lambda - \frac{1}{2\lambda}$ thus

$\varphi'(\lambda) = \frac{1}{\lambda} + \frac{1}{2\lambda^2}$.

(2) For the double Poisson distribution

$$P(X=x)=\frac{1}{2}\frac{e^{-m_1}m_1^x}{x!}+\frac{1}{2}\frac{e^{-m_2}m_2^x}{x!};\ 0,1,2,\ldots\ldots\ldots$$

Show that the estimator for m_1 and m_2 by the method of moment are $\mu_1' \pm \sqrt{\mu_2' - \mu_1' - (\mu_1')^2}$.

4 (a) Give the answer of following question : **4**

(1) Accepting H_0 when H_0 is false is ______ error.

(2) Probability of type I error is called _____.

(3) If β is the probability of type II error, the power of the test is ____.

(4) A null hypothesis is rejected if the value of a test statistics lies in the _____.

(b) Write any one : **2**

(1) Define MP test.

(2) Given a random sample $x_1, x_2, x_3, \ldots\ldots\ldots, x_n$ from the distribution with pdf $f(x,\theta)=\frac{1}{\theta}$; where $0 \le x \le \theta$.

Obtain power of the test for testing $H_0 : \theta = 1.5$ against $H_1 : \theta = 2.5$ where $c = \{x : x \ge 0.8\}$.

(c) Write any one : **3**

(1) Give a random sample $x_1, x_2, x_3, \ldots\ldots\ldots, x_n$ from distribution with p.d.f. $f(x;\theta)=\theta e^{-\theta x}$; $0 \le x \le \infty$, $\theta > 0$. Use the Neyman Pearson Lemma to obtain the best critical region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$.

(2) Let p be the probability that coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type-I error, type-II error and power of test.

(d) Write any one : **5**

(1) State and prove the Neyman-Pearson Lemma.

(2) Use Neyman-Pearson Lemma to obtain the best critical region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ in the case of normal distribution $N\left(\theta, \sigma^2\right)$ where σ^2 is known

5 (a) Give the answer of following question : **4**

(1) Likelihood ratio test for testing a hypothesis, simple or composite, against a _____ or ______ alternative hypothesis.

(2) Likelihood ratio test is relation the maximum ______ estimates.

(3) To decide about H_0, SPRT involves ______ regions.

(4) The decision criteria in SPRT depends on the function of _______ and ______ errors.

(b) Write any one : **2**

(1) Define UMP test.

(2) Define ASN function of SPRT.

(c) Write any one : **3**

(1) Construct SPRT of binomial distribution for testing $H_0 : \theta = p_0$ against $H_1 : \theta = p_1 (> p_0)$.

(2) If $x \sim N\left(\mu, \sigma^2\right)$. To likelihood ratio test $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \neq \sigma_0^2$.

(d) Write any one : **5**

(1) Let sample distribution $x_1, x_2, x_3, \ldots\ldots\ldots, x_n$ taken from

$$f\left(x, \theta\right) = \frac{1}{\sqrt{2\pi}} e^{-\left\{\frac{(x-\theta)^2}{2}\right\}} \text{ where } -\infty \leq x \leq \infty,$$

$-\infty \leq \theta \leq \infty$.

To likelihood ratio $H_0 : \theta \leq \theta_0$ against $H_1 : \theta \leq \theta_0$.

(2) Give the SPRT for test $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1 \left(> \lambda_0\right)$, in sampling from a Poission distribution. Also obtain its OC and ASN function.